

Math 115
Spring 2019
Lecture 28

? $a^2 + b^2 = c^2$?
 $y = mx + b$? $d = rt$

find only the excluded values. Use \neq
for final answer:

1) $\frac{3}{x+2} - \frac{5}{x} = 1$

$x+2 \neq 0$

$x \neq -2$

$x \neq 0$

E.V. $-2, 0$

2) $\frac{8}{(x-8)(x+2)} = \frac{7}{3x+10}$

$x-8 \neq 0$

$x \neq 8$

$x+2 \neq 0$

$x \neq -2$

$3x+10 \neq 0$

$x \neq -\frac{10}{3}$

3) $\frac{4}{x^2+8x-9} - \frac{1}{x^2-4} = 0$

$x^2+8x-9 \neq 0$

$(x+9)(x-1) \neq 0$

$x \neq -9$

$x \neq 1$

E.V. $-9, 1, \pm 2$

$x^2-4 \neq 0$

$(x+2)(x-2) \neq 0$

$x \neq -2, x \neq 2$

E.V.

$-2, -\frac{10}{3}, 8$

Solve:

Give LCD & E.V.

$$\textcircled{1} \frac{2x+1}{5} = \frac{7x+5}{15}$$

LCD=15, NO E.V.

$$15(2x+1) = 5(7x+5)$$

$$30x + 15 = 35x + 25$$

$$30x - 35x = 25 - 15$$

$$-5x = 10 \quad \boxed{x = -2} \quad \{-2\}$$

$$\textcircled{2} \frac{x}{x-4} = 5 + \frac{4}{x-4}$$

LCD = $x-4$, E.V. 4

$$x = 5(x-4) + 4$$

$$x = 5x - 20 + 4$$

$$x - 5x = -16$$

$$-4x = -16$$

$$\boxed{x = 4}$$

$$\textcircled{3} \frac{x}{4-x} = \frac{2}{x} \quad \text{LCD} = (4-x)x$$

E.V. 4, 0

$$x \cdot x = 2(4-x)$$

$$x^2 = 8 - 2x$$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

$\{0, -4\}$

$$x = -4$$

$$x = 2$$

$$\textcircled{4} \frac{3}{x-5} - \frac{2}{x+5} = \frac{20}{x^2-25}$$

LCD = $(x-5)(x+5)$

E.V. ± 5

$$3(x+5) - 2(x-5) = 20$$

$$3x + 15 - 2x + 10 = 20$$

$$x + 25 = 20$$

$$x = -5$$

\emptyset

$$\textcircled{5} \frac{5}{x-2} + \frac{10}{x+2} = 7$$

LCD = $(x-2)(x+2)$

E.V. ± 2

$$5(x+2) + 10(x-2) = 7(x-2)$$

$$5x + 10 + 10x - 20 = 7(x-2)$$

$$15x - 10 = 7x^2 - 28$$

$$7x^2 - 28 - 15x + 10 = 0$$

$$7x^2 - 15x - 18 = 0$$

$$a=7 \quad b=-15 \quad c=-18 \quad b^2 - 4ac = (-15)^2 - 4(7)(-18)$$

$$= 225 + 504 = 729$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{15 \pm \sqrt{729}}{14}$$

when $b^2 - 4ac$ is a perfect sq, Trinomial is factorable

$$\textcircled{6} \quad \frac{x}{x^2+x-2} + \frac{x}{x^2-1} = \frac{x}{x^2+3x+2}$$

LCD = $(x+2)(x-1)(x+1)$

$$\frac{x}{(x+2)(x-1)} + \frac{x}{(x-1)(x+1)} = \frac{x}{(x+2)(x+1)}$$

E.V. -2, ±1

$$x(x+1) + x(x+2) = x(x-1)$$

$$x^2+x + \cancel{x^2} + 2x = \cancel{x^2} - x$$

$$x^2 + 3x + x = 0$$

$$x^2 + 4x = 0$$

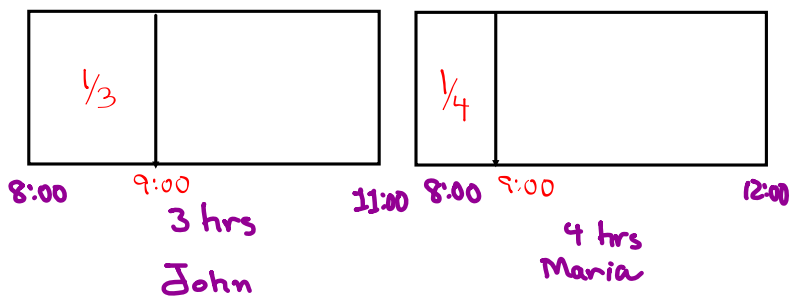
$$x(x+4) = 0$$

$$x = 0$$

$$x = -4$$

$$\{-4, 0\}$$

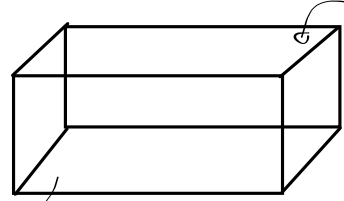
Work Problem



Both working together

work by John	+	work by Maria	=	one Complete work
Rate · time		Rate · time		
$\frac{1}{3} \cdot t$		$+\frac{1}{4} \cdot t = 1$		
				$\frac{t}{3} + \frac{t}{4} = 1$
				LCD = 12
				$4t + 3t = 12$
				$7t = 12$
				$t = \frac{12}{7}$
				(t = 1.71 hrs)

Pipe A can fill up an empty pool in 8 hrs.
 Pipe B can empty a full pool in 10 hrs.
 If the pool is empty, and both pipes working,
 how long does it take to fill up the pool?



Pipe A
 8 hrs
 +
 Pipe B
 10 hrs
 -

work by Pipe A — work by Pipe B = one complete work

$$\frac{1}{8} \cdot t - \frac{1}{10} \cdot t = 1$$

LCD = 40

$$5t - 4t = 40$$

$t = 40$

40 hrs

It takes John 3 hrs longer than Maria to complete a job. Together, they can finish the job in 21 hrs.
 How long does it take each one to do the job alone?

Maria $\rightarrow x$ hrs \rightarrow Rate $\frac{1}{x}$
 John $\rightarrow x+3$ hrs \rightarrow Rate $\frac{1}{x+3}$

work by Maria + work by John = one complete work

$$\frac{1}{x} \cdot 21 + \frac{1}{x+3} \cdot 21 = 1$$

Solve $\frac{21}{x} + \frac{21}{x+3} = 1$

LCD = $x(x+3)$
 E.V. 0, -3
 $\Rightarrow x > 0$

$$21(x+3) + 21x = x(x+3)$$

$$21(x+3) + 21x = x(x+3)$$

$$21x + 63 + 21x = x^2 + 3x$$

$$42x + 63 = x^2 + 3x$$

$$x^2 + 3x - 42x - 63 = 0$$

$$x^2 - 39x - 63 = 0$$

$$a=1 \quad b=-39 \quad c=-63$$

$$b^2 - 4ac = (-39)^2 - 4(1)(-63)$$

$$= 1773$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{39 \pm \sqrt{1773}}{2}$$

$$\approx \frac{39 \pm 42}{2}$$

$$x = \frac{39+42}{2} = 40.5$$

$$x = \frac{39-42}{2} = -1.5$$

Maria \rightarrow 40.5 hrs
 John \rightarrow 43.5 hrs

John can wash the car 18 minutes less than Maria can.
 Together, they can do it in 12 minutes.
 How long alone?

Maria \rightarrow x \rightarrow Rate: $\frac{1}{x}$
 John \rightarrow $x-18$ \rightarrow Rate: $\frac{1}{x-18}$

$$\frac{1}{x} + \frac{1}{x-18} = 1$$

LCD = $x(x-18)$
 E.V. 0, 18
 $\Rightarrow x > 18$

$$\frac{1}{x} \cdot 12 + \frac{1}{x-18} \cdot 12 = 1$$

$$12(x-18) + 12(x) = 1 \cdot x(x-18)$$

$$12x - 216 + 12x = x^2 - 18x$$

$$24x - 216 = x^2 - 18x$$

$$x^2 - 18x - 24x + 216 = 0$$

$$x^2 - 42x + 216 = 0$$

$$(x-36)(x-6) = 0$$

36
 Maria 36 minutes
 John 18 minutes

John and Maria can do a job together in 5 days.

Maria can do it alone in 7 days.

How long does it take John to do it alone?

Maria \rightarrow 7 Days \rightarrow Rate: $\frac{1}{7}$

John \rightarrow x Days \rightarrow Rate: $\frac{1}{x}$

$$\frac{1}{7} \cdot 5 + \frac{1}{x} \cdot 5 = 1$$

$$\frac{5}{7} + \frac{5}{x} = 1$$

$$\text{LCD} = 7x$$

$$\cancel{7x} \cdot \frac{5}{\cancel{7}} + \cancel{7x} \cdot \frac{5}{\cancel{x}} = 7x \cdot 1$$

$$5x + 35 = 7x$$

$$7x - 5x = 35$$

$$2x = 35$$

$$x = 17.5$$

17.5 Days

The sum of reciprocal of two consecutive integers is $\frac{9}{20}$. Find all such integers

$x, x+1$

$$\frac{1}{x} + \frac{1}{x+1} = \frac{9}{20}$$

$$\text{LCD} = 20x(x+1)$$

$$20(x+1) + 20x = 9x(x+1)$$

$$20x + 20 + 20x = 9x^2 + 9x$$

$$40x + 20 = 9x^2 + 9x$$

$$9x^2 + 9x - 40x - 20 = 0$$

$$9x^2 - 31x - 20 = 0$$

$$\cancel{x = \frac{-5}{9}}$$

$$x = 4$$

4 & 5

Not integer

Find two consecutive even integers such that the difference of their reciprocals is $\frac{1}{40}$.

$$\boxed{\frac{1}{x}} - \boxed{\frac{1}{x+2}} = \boxed{\frac{1}{40}}$$

$\rightarrow x, x+2$
 $\frac{1}{x}, \frac{1}{x+2}$
8, 10
-10, -8

LCD = $40x(x+2)$

$$40(x+2) - 40x = x(x+2)$$

$$\cancel{40x} + 80 - \cancel{40x} = x^2 + 2x$$

$$x^2 + 2x - 80 = 0$$

$\rightarrow x = 8$
 $\rightarrow x = -10$

Linear Motion:

You drive @ 40 mph for 2.5 hrs, the

distance you travel is

$\frac{40 \text{ miles}}{1 \text{st hr}}$	$\frac{40 \text{ miles}}{2 \text{nd hr}}$	$\frac{20 \text{ miles}}{.5 \text{ hr}}$
Total distance		
100 miles.		

$$d = r t$$

d → distance
 r → Rate (speed)
 t → time

$$d = 40(2.5)$$

$$= 100 \text{ miles.}$$

$$d = r t$$

Solve for t

$$\frac{d}{r} = t$$

You need to travel 750 miles in 12 hrs.
Find average speed.

$$d = r t$$

$$\frac{d}{t} = r \quad \frac{750}{12} = r \rightarrow r = 62.5$$

62.5 mph

How long does it take to travel 450 miles at 60 mph?

$$d = r t$$

$$\frac{d}{r} = t$$

$$\frac{450}{60} = t$$

$$t = 7.5$$

7.5 hrs

John traveled 150 miles in the same time that Maria traveled 210 miles.

John was driving 20 mph slower than Maria.

Find speed for both.

$$t_{\text{John}} = t_{\text{Maria}}$$

$$\frac{5 \cancel{15} 150}{x-20} = \frac{\cancel{21} 7 210}{x}$$

$$7(x-20) = 5x$$

	d	r	t
Maria	210	x	t
John	150	x-20	t

$$\rightarrow 7x - 140 = 5x$$

Maria
70mph

$$7x - 5x = 140$$

John
50mph

$$2x = 140$$

$$x = 70$$

John drove 200 miles in the same time that Maria drove 140 miles.

John was driving 15 mph faster than Maria.

Find speed for both.

$$t_{\text{John}} = t_{\text{Maria}}$$

$$\frac{200}{x+15} = \frac{140}{x}$$

	d	r	t
John	200	$x+15$	t
Maria	140	x	t

$$10x = 7(x+15)$$

$$10x = 7x + 105$$

$$3x = 105$$

$$x = 35$$

Maria 35 mph
John 50 mph.

Maria drove 90 miles in the city.

John drove 130 miles on the highway.

John drove 20 mph faster than Maria.

Total time 4 hrs. Find speed for both.

	d	r	t
John	130	$x+20$	t_1
Maria	90	x	t_2

$$t_1 = \frac{130}{x+20} \quad t_2 = \frac{90}{x}$$

$$t_1 + t_2 = 4$$

$$\frac{130}{x+20} + \frac{90}{x} = 4$$

$$\text{LCD} = x(x+20)$$

$$130x + 90(x+20) = 4x(x+20)$$

$$130x + 90(x+20) = 4x(x+20)$$

$$130x + 90x + 1800 = 4x^2 + 80x$$

$$220x + 1800 = 4x^2 + 80x$$

$$4x^2 + 80x - 220x - 1800 = 0$$

$$4x^2 - 140x - 1800 = 0$$

Divide by 4 to reduce

$$x^2 - 35x - 450 = 0$$

$$(x + 10)(x - 45) = 0$$

$$\rightarrow x = -10$$

$$x = 45$$

Maria 45 mph

John 65 mph

See 20 for Tuesday